Erratum: Classical specific heat of an atomic lattice at low temperature, revisited [Phys. Rev. E 57, 100 (1998)]

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The last paragraph on page 102, beginning with ''In a canonical ensemble . . . ,'' and ending on page 103 with '' . . . the temperature of the system,'' gives a correct result, but the derivation is wrong. The whole paragraph should be replaced with the following.

In a canonical ensemble the probability of a state is given by

$$P(\vec{x}) = \frac{e^{-\beta H(\vec{x})}}{\int_\Gamma e^{-\beta H(\vec{x})} d\vec{x}} , \tag{4}$$

where H is the Hamiltonian, $\vec{x}$ a point of the phase space Γ, and $\beta = (k_B T)^{-1}$. If the system can be decomposed into M small subsystems with negligible interaction, one has $H(\vec{x}) \simeq \Sigma_{j=1}^{M} H_j(\vec{x}_j)$, where H_j is the Hamiltonian of the subsystem described by a set of coordinates and momenta $\vec{x}_j$. The distribution law of the whole system can be expressed by the combination of the distribution laws of the (almost) independent subsystems:

$$\frac{e^{-\beta H(\vec{x}_1, \ldots, \vec{x}_M)}}{\int_\Gamma e^{-\beta H(\vec{x}_1, \ldots, \vec{x}_M)} d\vec{x}_1 \cdots d\vec{x}_M} = \prod_{j=1}^{M} \frac{e^{-\beta H_j(\vec{x}_j)}}{\int_\Gamma e^{-\beta H_j(\vec{x}_j)} d\vec{x}_j} .$$

This means that the canonical probability at temperature T of a state of the system is the product of the independent probabilities of the M subsystems at the same temperature. It is as if each subsystem were individually in contact with the thermal bath, which determines the temperature of the system. If the system has constant energy but all subsystems are small, the total energy is not an effective constraint for the energy of each subsystem; with the probability distribution of the latter being ''almost identical'' to the canonical case, one can consider all subsystems to interact individually with a sort of thermal bath. Given the equivalence of the canonical and the microcanonical probability-distribution laws of small subsystems, we can assume a canonical behavior in our system for each group of normal modes, which is small in comparison with the whole set of normal modes.

